MATHEMATICS

K-12 ACADEMIC CONTENT STANDARDS

INTRODUCTION

CATALINA FOOTHILLS SCHOOL DISTRICT

A 21st Century Learning Community

Overview of the Mathematics Standards

The Catalina Foothills School District (CFSD) Standards for Mathematics are aligned to the 2010 Arizona Mathematics Standards, which were adopted by the State Board of Education on June 8, 2010, and the CFSD Governing Board on June 8, 2011. The standards are articulated from kindergarten to high school, created to ensure that all students are college and career ready in mathematics no later than the end of high school.

In K-8 (Kindergarten, Elementary, and Middle School) the content for each *grade* is organized by *measurement topics / domains*, as described in the table below. For example: In Grade 1, the content includes Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry.

<table>
<tr><th>Grade</th><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS Conceptual Categories</th></tr>
<tr><td rowspan="6">Measurement Topics / Domains</td><td>Counting & Cardinality</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">Ratios & Proportional Relationships</td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3" rowspan="2">The Number System</td><td rowspan="2">Number & Quantity</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Fractions</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics & Probability</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
</table>

In High School, the standards are arranged in *conceptual categories*, such as Algebra or Functions. In each conceptual category there are *domains*, such as Creating Equations and Interpreting Functions.

Conceptual Category	Number & Quantity	Algebra	Functions	Geometry	Statistics & Probability
Measurement Topics / Domains	The Real Number System	Seeing Structure in Expressions	Interpreting Functions	Congruence	Interpreting Categorical & Quantitative Data
	Quantities	Arithmetic with Polynomials & Rational Expressions	Building Functions	Similarity, Right Triangles, & Trigonometry	Making Inferences & Justifying Conclusions
	The Complex Number System	Creating Equations	Linear, Quadratic, & Exponential Models	Expressing Geometric Properties with Equations	Conditional Probability & the Rules of Probability
	Vector & Matrix Quantities	Reasoning with Equations & Inequalities	Trigonometric Functions	Geometric Measurement & Dimension	Using Probability to Make Decisions

Consistent with the CFSD curriculum framework, the *domains* of mathematics, as represented in the diagrams above, *are* the CFSD measurement topics (categories of learning) that further define the level- or course-specific standards. The grade level standards define what students will know and be able to do, and are aligned to each measurement topic or domain of mathematics. These standards provide a focus for instruction and assessment each year.

Mathematical Practices

The Standards for Mathematical Practice describe characteristics and traits that mathematics educators at all levels develop in their students. The Standards for Mathematical Practices are taught in conjunction with the course- and grade level-specific content standards across K-12. The eight Mathematical Practices are as follows:

Habits of a Productive Mathematical Thinker:

- Make sense of problems and persevere in solving them.
- Attend to precision.

Reasoning and Explaining

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

Modeling and Using Tools

- Model with mathematics.
- Use appropriate tools strategically.

Seeing Structures and Generalizing

- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

In addition to mastering academic content, our students will need to develop a broader set of skills to thrive in a rapidly evolving, technology-saturated world. To this end, CFSD has formally integrated seven (7) deep learning proficiencies (formerly called 21st century skills) and educational technology within the curriculum, where appropriate. Frameworks have been created at each level to guide planning, instruction, and assessment of academic content and the deep learning proficiencies. We want to develop highly engaged students who are self-regulated critical thinkers.

CFSD's deep learning proficiencies are as follows: Critical Thinking and Problem Solving, Citizenship, Communication, Collaboration, Creativity and Innovation, and Systems Thinking. Our expectations for development 21st century learners are clearly defined in the CFSD 21st Century Skill Rubrics (See rubrics in the elementary, middle, or high school program of study on this website). We intend to measure our students' growth in these skills.

Required Fluencies in the Standards for Mathematics

When it comes to measuring the full range of the Standards, usually the first things that come to mind are the mathematical practices, or perhaps the content standards that call for conceptual understanding. However, the Standards also address another aspect of mathematical attainment that is seldom measured at scale either: namely, whether students can perform calculations and solve problems quickly and accurately. At each grade level in the Standards, one or two fluencies are expected:

Grade	Required Fluency
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20[1] Add/subtract within 100 (pencil and paper)
3	Multiply/divide within 100[2] Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x+q) = r$
8	Solve simple 2x2 systems by inspection

Fluent in the Standards means "fast and accurate." It might also help to think of fluency as meaning the same thing as when we say that somebody is fluent in a foreign language: when you're fluent, you flow. Fluent isn't halting, stumbling, or reversing oneself. Assessing fluency requires attending to issues of time (and even perhaps rhythm, which could be achieved with technology).

The word *fluency* was used judiciously in the Standards to mark the endpoints of progressions of learning that begin with solid underpinnings and then pass upward through stages of growing maturity. In fact, the rarity of the word itself might easily lead to fluency becoming invisible in the Standards—one more among 25 things in a grade, easily overlooked. Assessing fluency could remedy this, and at the same time allow data collection that could eventually shed light on whether the progressions toward fluency in the Standards are realistic and appropriate.

[1] By end of year, know from memory all sums of two one-digit numbers
[2] By end of year, know from memory all products of two one-digit numbers

Instructional Shifts for Implementing the Mathematics Standards

There are three key shifts associated with the mathematics standards.

Focus — The goal is to focus strongly where the standards focus. The curriculum significantly narrows and deepens the way time and energy are spent in the mathematics classroom. The standards focus deeply on the major work of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the mathematics classroom.

Coherence — Coherence is connecting ideas across grades, and linking to major topics within grades. The standards are designed around coherent progressions from grade to grade. Students build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning. Instead of allowing additional or supporting topics to detract from the focus of the grade, these topics can serve the grade-level focus.

Rigor — In major topics, conceptual understanding, procedural skill and fluency, and application are pursued with equal intensity.

- Emphasis is placed on conceptual understanding of key concepts, such as place value and ratios. Teachers support students' ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures.

- Students build speed and accuracy in calculation. Teachers structure class time and/or homework time for students to practice core functions, such as single-digit multiplication, so that they have access to more complex concepts and procedures.

Students use math flexibly for applications. Teachers provide opportunities for students to apply math in context. Teachers in content areas outside of math, particularly science, ensure that students are using math to make meaning of and access content.